3-D Inductance and Resistance Extraction for Interconnects

Shuzhou Fang, Liu Yang and Zeyi Wang

Dept. of Computer Science & Technology
Tsinghua University, Beijing 100084, China
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Content

- PEEC & Multipole Acceleration (MPA)
  - Review
  - Recent work
    - filament refinement in MPA
    - investigation of preconditioned GMRES
  - Numerical results
- A New Extraction Model -- 3-D Minimum-Order Boundary Integral Equation Method
  - Previous methods
  - Our method
  - Numerical results
  - Conclusions
- Published Papers
**PEEC & MPA — Review**

- **Model**
  - PEEC (Partial Element Equivalent Circuit)

- **Flow Chart**
  
  1. Read input files
  2. Geometrical partition
  3. Discretization of formulation
  4. Preconditioning
  5. Solve equation with GMRES
  6. Output results
PEEC & MPA — Review

- Feature
  - Convenient description with ability to describe trapezoid shape on top view
  - Automatically partitioning filaments in consideration of skin effect
  - Multipole acceleration based on non-uniform cube subdivision
  - GMRES preconditioned in solving set of linear equations
Multipole acceleration

Cube subdivision

- Uniform cube subdivision
- Non-uniform cube subdivision
- Modified non-uniform cube subdivision

(a) Particles distribution
(b) Uniform
(c) Non-uniform
(d) Modified non-uniform
Filament refinement

When the length of a filament is two or more times larger than the width of cube which the filament belongs to, the filament should be subdivided twice in current flowing direction.
Investigation of Preconditioned GMRES

- Jacob preconditioning (Jacob)
  - Inverse diagonal of impedance matrix \( Z \)

- Local inversion preconditioning (INV)
  - Inverse block diagonal of impedance matrix \( Z \) approximately

- Incomplete LU factorization preconditioning (ILU)
  - Factorize approximation of impedance matrix \( Z \)
  - Incomplete LU factorization method of level zero
Filament refinement

30 pin structure
### Result

<table>
<thead>
<tr>
<th></th>
<th>FastHenry</th>
<th>FIE</th>
<th>FIE (Filament refinement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknowns</td>
<td>1820</td>
<td>1820</td>
<td>1760</td>
</tr>
<tr>
<td>Total time</td>
<td>245.2s</td>
<td>40.91s</td>
<td>14.22s</td>
</tr>
<tr>
<td>Result</td>
<td>26.39nH</td>
<td>24.74nH</td>
<td>25.96nH</td>
</tr>
<tr>
<td>Speed up</td>
<td></td>
<td>5.99</td>
<td>17.24</td>
</tr>
<tr>
<td>Discrepancy</td>
<td></td>
<td>6.25%</td>
<td>1.64%</td>
</tr>
</tbody>
</table>

Frequency: 1GHz
### 60 pin structure

<table>
<thead>
<tr>
<th></th>
<th>FastHenry</th>
<th>FIE</th>
<th>FIE (Filament refinement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknowns</td>
<td>3640</td>
<td>3640</td>
<td>3520</td>
</tr>
<tr>
<td>Total time</td>
<td>656.7s</td>
<td>70.56s</td>
<td>31.7s</td>
</tr>
<tr>
<td>Result</td>
<td>26.39nH</td>
<td>24.72nH</td>
<td>25.96nH</td>
</tr>
<tr>
<td>Speed up</td>
<td></td>
<td>9.31</td>
<td>20.72</td>
</tr>
<tr>
<td>Discrepancy</td>
<td></td>
<td>6.33%</td>
<td>1.64%</td>
</tr>
</tbody>
</table>

Frequency: 1GHz
Investigation of Preconditioned GMRES

- CPU time of three preconditioned GMRES Jacob, INV and ILU compared with non-preconditioned GMRES for three examples Simple, Vias and 30pin provided by FastHenry

Simple structure

Vias structure
# PEEC & MPA – Numerical results

<table>
<thead>
<tr>
<th></th>
<th>Unknowns</th>
<th>Non-precondition</th>
<th>Jacob</th>
<th>INV</th>
<th>ILU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>128</td>
<td>0.20s</td>
<td>0.18s</td>
<td>0.17s</td>
<td><strong>0.07s</strong></td>
</tr>
<tr>
<td>Vias</td>
<td>372</td>
<td>2.68s</td>
<td>2.02s</td>
<td>1.28s</td>
<td><strong>0.67s</strong></td>
</tr>
<tr>
<td>30pin</td>
<td>455</td>
<td>6.49s</td>
<td>4.44s</td>
<td>3.69s</td>
<td><strong>3.49s</strong></td>
</tr>
</tbody>
</table>

**Frequency:** 1GHz
CPU time curves of three preconditioned GMRES Jacob, INV, ILU and non-preconditioned GMRES versus number of unknowns for 30 pin structure.

Frequency: 1GHz
New Model – Previous methods

1. Finite Volume Method (PEEC, FastHenry)
   - Volume discretization of the conducting regions -> filaments
   - Current direction along axis of the filament (no current crossing the filament sides)

Professor Li Zhengfan et al proposed an improved PEEC model in 2000

Professor Pileggi et al proposed a new model using the vector magnet potential and scale potential in 1999
New Model – Previous methods

2. Direct Boundary Element Method
(Junfeng Wang, DAC99)

- Only deal with the conductor surfaces
- At least 6 unknowns at each node over conductor surface (only extracting RL)
New Model – Key points of our method

- In conductor region, based on the idea of Minimum Order BEM, obtain the surface expression of MVP (Magnetic Vector Potential)

- In non-conducting region, applying SOVP concept to express the solution of MVP, then obtain the surface expression

- Substitute the surface expressions of MVP into boundary conditions to get the new boundary integral equations
New Model: Key points of our method

\(1\) \( t^k(\xi) \cdot (\mu \int_{S_i} K(\xi, \eta) J_i(\eta) \, dS_i - A^s_i(\xi)) + \int_{S_{all}} e \times \nabla G(\xi, \eta) \sigma_0(\eta) \, dS - \frac{\sigma_0(\xi) e \times n(\xi)}{4} = 0, k = 1, 2 \)

\(2\) \( n(\xi) \cdot (\mu \int_{S_i} K(\xi, \eta) J_i(\eta) \, dS_i - A^s_i(\xi)) = 0 \)

\(3\) \( \int_{l_i} \nabla \times A_i \cdot dI_i = \mu I_i \)
New Model – Key points of our method

At each point on the conductor surface, there are 3 unknowns:

★ 2 components of virtual surface current density $J$
★ 1 virtual charge density

In each straight conductor:

★ there is 1 extra unknown
★ the number of the conductor is much less than the number of the elements
New Model – Numerical results

1. One Conductor with Round Cross Section

- Conductivity: $3 \times 10^7$ S/m
- Radius: 1 cm  Current: 60Hz  1000A

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretization (perimeter x axis)</td>
<td>$8 \times 8$</td>
<td>$8 \times 16$</td>
<td>$8 \times 24$</td>
<td>$8 \times 32$</td>
<td>$8 \times 40$</td>
</tr>
<tr>
<td>Our result ($kA/m^2$)</td>
<td>2326</td>
<td>2635</td>
<td>2994</td>
<td>3178</td>
<td>3248</td>
</tr>
<tr>
<td>Analytical result ($kA/m^2$)</td>
<td></td>
<td></td>
<td></td>
<td>3265 (infinite length)</td>
<td></td>
</tr>
</tbody>
</table>

- When the length increases, the result approaches the analytical result
- The difference is within 0.4% when the length is 25 cm
New Model – Numerical results

2. Two Orthogonal Conductors

- Conductivity: $3 \times 10^7$ S/m
- Radius: 0.1 µm
- Length: 0.5 µm
New Model – Numerical results

- Spacing: 0.1 µm  Frequency : 1GHz

<table>
<thead>
<tr>
<th>(Z_{ij})</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.75 e-4 + 9.52 e-3 j</td>
<td>7.14 e-7 - 2.46 e-7 j</td>
</tr>
<tr>
<td>2</td>
<td>7.14 e-7 - 2.46 e-7 j</td>
<td>8.75 e-4 + 9.52 e-3 j</td>
</tr>
</tbody>
</table>

- The mutual resistance is about 0.08% of self-resistance.
- This is why we can ignore the mutual impedance when the frequency is relatively low and spacing relatively wide.
- FastHenry always gives zero mutual impedance of two orthogonal conductors, no matter how high the frequency is applied.
New Model — Numerical results

- Spacing: 0.05 µm  Frequency: 10GHz

<table>
<thead>
<tr>
<th>$Z_{i,j}$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.93 \times 10^{-2}$ + $7.32 \times 10^{-2} j$</td>
<td>$7.46 \times 10^{-4}$ + $7.57 \times 10^{-4} j$</td>
</tr>
<tr>
<td>2</td>
<td>$7.46 \times 10^{-4}$ + $7.57 \times 10^{-4} j$</td>
<td>$2.93 \times 10^{-2}$ + $7.32 \times 10^{-2} j$</td>
</tr>
</tbody>
</table>

- The mutual resistance is about 2.55% of self-resistance
- A conductor may cross many other conductors, the induced voltage is the algebraic sum of products of mutual impedance and current, and cannot be ignored any more, even if they are all orthogonal to the first conductor.
Michael W. Beattie & Lawrence T. Pileggi showed that there were mutual impedance between orthogonal lines ("Electromagnetic parasitic extraction via a multipole method with hierarchical refinement," IEEE/ACM International Conference on Computer-Aided Design, Digest of Technical Papers, p 437-444, Nov 7-11, 1999)
Avoiding volume discretization of the conducting regions

As an indirect boundary element method, it has the least number of unknowns

Can calculate the mutual RL between orthogonal conductors
Published Papers

- Shuzhou Fang and Zeyi Wang, “A 3-D Minimum-order Boundary Integral Equation Technique to Extract Frequency-dependent Inductance and Resistance in VLSI,” accepted by Science in China
- “3-D ”
Thank you